Higher Dimensional Taub-NUTs and Taub-Bolts in Einstein-Maxwell Gravity

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Abstract

We present a class of higher dimensional solutions to Einstein-Maxwell equations in d–dimensions. These solutions are asymptotically locally flat, de–Sitter, or anti–de Sitter space-times. The solutions we obtained depend on two extra parameters other than the mass and the nut charge. These two parameters are the electric charge q and the electric potential at infinity, V, which has a non-trivial contribution. We Analyze the conditions one can impose to obtain Taub-Nut or Taub-Bolt space-times, including the four-dimensional case. We found that in the nut case these conditions coincide with that coming from the regularity of the one-form potential at the horizon. Furthermore, the mass parameter for the higher dimensional solutions depends on the nut charge and the electric charge or the potential at infinity.

1 Introduction

The importance of Taub-Nut solutions covers a wide area of application that extends from general relativity to string theory. These solutions [1] are characterized by non-vanishing nut charges. As a result, they are locally asymptotically flat, i.e., their boundaries are not $S^1 \times S^2$ but S^1 fiber over S^2. These boundaries are topologically interesting since they do have a non-vanishing first Chern number N which is proportional to the nut charge. In these space-times one can not define a global time function, therefore, these manifolds can not be foliated using constant time surfaces and it is not possible to describe their time evolution through a unitary Hamiltonian evolution. Taub-Nut spaces are also characterized by the existence of zero-dimensional fixed point set of the $U(1)$ isometry generated by the time-like Killing vector ∂_τ, which is called a nut. This is in contrast with the higher-dimensional fixed-point set appears in spherically symmetric black hole solutions, which is called a bolt. Tau-Nut solutions also possess a singular one dimensional string analogous to Dirac string [3]which is called Misner string. The singularity of Misner string is a coordinate singularity, since one fails to describe the manifold using a single coordinate system. Misner string contributes to the entropy of the black hole, therefore the entropy is not simply one quarter the area of the horizon. This is a very interesting

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property in gravitation, which reveals some features of the geometric entropy and has been studied by Hawking and Hunter [7, 6, 5].

Another attractive feature of the Taub-Nut solution is the celebrated Gross-Perry-Sorkin monopole[4]. This monopole can be constructed by adding another trivial dimension to get a five dimensional metric followed by Kaluza Klein compactifing the theory to four-dimensional along the Euclidian time direction. As a result, one gets a singular four-dimensional monopole solution. This shows how perfectly regular higher dimensional solutions can give rise to a singular lower dimensional one.

Taub-Nut solution plays an important role in revealing the nature of D6-branes in type IIA string theory and its relation to M-theory(for a review see *e.g.*[8]). Probing the background geometry of N coincident D6-branes using D2-branes shows that the transverse dimensions to both branes are not three, but in fact four dimensions and the metric is the Taub-Nut metric . The extra dimension is the Euclidian time direction which is compact. It comes from the gauge degrees of freedom of the $U(1)$ gauge theory living on the D2-brane world-volume. This shows that the D6-brane is a Kaluza-Klein monopole from the M-theory prospective. Another interesting application of Nut charged space-times is to extend and test the validity of the AdS/CFT correspondence to locally asymptotically AdS solutions, and study their thermodynamics[13, 15, 14].

Therefore, it is interesting to find new Taub-Nut and Taub- bolt solutions in higher dimensions with an eye on possible applications of these results in various supergravity/string theories. Here we present a class of higher dimensional $U(1)$ electrically charged solutions with nut charge, which are generalizations to the solutions found in [11, 15] (see also [16] for discussions on their thermodynamics).

The action for Einstein-Maxwell theory in d–dimensions for asymptotically (anti)–de–Sitter spacetimes is

$$I_d = -\frac{1}{16\pi G_d}\int_{\mathcal{M}} d^d x\sqrt{-g}\left(R + \frac{(d-1)(d-2)}{l^2} - F^{\mu\nu}F_{\mu\nu}\right), \tag{1.1}$$

where $\Lambda = \frac{-(d-1)(d-2)}{2l^2}$ is the cosmological constant, G_d is Newton's constant in d–dimensions, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, where A_μ is the vector potential. Varying the action with respect to the metric and the vector field A_μ we get the following field equations

$$\begin{aligned} G_{\mu\nu} - \Lambda g_{\mu\nu} &= 2\,T_{\mu\nu}, \\ \partial_\mu\left(\sqrt{-g}F^{\mu\nu}\right) &= 0, \end{aligned} \tag{1.2}$$

where

$$T_{\mu\nu} = F_{\mu\rho}\,F^{\rho}_{\nu} - \frac{1}{4}g_{\mu\nu}F^{\mu\nu}F_{\mu\nu}. \tag{1.3}$$

Here we consider only the Euclidian sections of these metrics. To go to the Lorentzian sections one can analytically continue the coordinate τ and the parameter n (i.e., $\tau \to i\,t$, $q \to i\,q$, $V \to i\,V$ and $n \to i\,n$). The general form of the Taub-Nut/Bolt metric is given by

$$ds^2 = F(r)^{-1}dr^2 + (r^2 - n^2)\,d\Sigma^2_{\mathcal{B}} + F(r)(d\tau + \mathcal{A})^2 \tag{1.4}$$

where the metric $d\Sigma^2_{\mathcal{B}}$ is over an even dimensional Einstein-Kähler manifold $\mathcal{B}$, $F(r)$ is some function of r. n is called the "Nut charge" and $\mathcal{A}$ is the potential of the Kähler form $\mathcal{F}$

$$\mathcal{F} = d\mathcal{A} \tag{1.5}$$

The hypersurface defined by $r = constant$ is a $U(1)$ fiber over the base manifold $\mathcal{B}$. The base space $\mathcal{B}$ can be any even dimensional Einstein-Kähler manifold. The case when $\mathcal{B} = \mathbb{CP}^n$ is special, since the

solution obtained is a non-singular space. Any other base space will be singular and therefore it can not be considered as a gravitational instanton. On the other hand these solutions are fine when they describe Bolt solutions, since these singularities will be hidden behind horizons. In the $\mathbb{CP}^n$ case the resulting hypersurfaces are squashed S^{d-1} and n here measures the amount of squashing.

In order to generalize this construction by giving electric charges to these solutions, we need some ansatz for the one-form potential A. We are going to adopt the following ansatz for the potential

$$A = g(r)\,(d\tau + \mathcal{A}) \tag{1.6}$$

The general solution for $g(r)$ depends on two parameters, one is the electric charge, q and the other is V, which can be viewed as potential at infinity; As $r \to \infty$ we have

$$A = (q/r^{d-3} + V)\,(d\tau + \mathcal{A}). \tag{1.7}$$

V plays an important role in studying the thermodynamics of Riessner-Nordstrom (A)dS solutions as has been revealed in [19]. Requiring regularity of the one-form potential A at the horizon relates the two parameters V and q. In fact we will not discuss the de–Sitter versions of these solutions, but they can be obtained, simply by taking $l \to i\,l$ which will be left for future work. While writing this article the work of [20] has been posted. The authors of [20] discuss similar solutions to the one presented here, but one can spot two general differences; i) the solutions presented here are a bit more general since it has two extra parameters V and q, ii) here we also discussed the conditions of having Taub-Nut or Taub-Bolt solutions and we have showed the possibility of having a Taub-Nut for certain values of the mass parameter and V. This article is organized as follows; in the first section we discuss conditions for obtaining nut and bolt solutions for asymptotically locally flat or locally AdS space-times in four dimensions. In the following section we present our results in six dimensions which can be generalized to eight and ten dimensions for asymptotically flat or AdS space-times. Finally we put some final remarks on the relevance of these solutions.

2 Four-Dimensional Solutions

2.1 Charged Taub-Nut-(A)dS Solution

The first electrically charged four-dimensional Taub-Nut solution has been introduced in [2]. More general versions of this solution have been studied in e.g., [9, 10]. Their supersymmetry has been discussed in [9], where the authors have showed that these solutions do preserve some supersymmetry for certain choices of their parameters. Here we present an (A)dS version of this solution with the additional parameter, V, that we mentioned in the introduction[1]. The charged Taub-Nut (Anti)-de-Sitter space version of the Brill's solution has the form

$$ds^2 = f(r)\,(d\tau - 2n\cos\theta\, d\phi)^2 + \frac{dr^2}{f(r)} + (r^2 - n^2)(d\theta^2 + sin\theta^2\, d\phi^2) \tag{2.8}$$

where $f(r)$ is given by

$$f(r) = \frac{r^4 + (l^2 - 6n^2)\,r^2 - 2m\,l^2\,r - l^2\,(q^2 + 4q\,V\,n\,k + 4\,n^2\,V^2\,(k^2-1) - n^2) - 3n^4}{l^2(r^2 - n^2)}. \tag{2.9}$$

The gauge potential has the form

$$A = g(r)\,(d\tau + 2n\,\cos\theta d\phi). \tag{2.10}$$

[1] As it will be clear from the discussion below this solution is equivalent to some of the solutions discussed in [9]

Here $g(r)$ is given by

$$g(r) = -V\,\frac{r^2+n^2+2\,n\,k\,r}{r^2-n^2} - q\,\frac{r}{r^2-n^2}, \tag{2.11}$$

q is the electric charge, m is the mass parameter, k is some constant[2] and V is the potential at infinity.

In order for this solution to describe a nut, we must have $f(r = n) = 0$, so that all of the extra dimensions collapse to zero size at the fixed-point set of the Killing vector, ∂_τ. This only happens at a specific value of the mass parameter, m, and the parameter q, namely

$$m_n = n - 4n^3/l^2 \tag{2.12}$$

$$q = -2n\,V\,(1+k) \tag{2.13}$$

otherwise the solution will have a horizon radius $r_h > n$ and become a bolt. Amusingly the last condition, i.e., $q = -2n\,V\,(1+k)$ can be obtained from requiring the regularity of the one-form A at the horizon as well. In fact this is might be a sign of consistency for including the extra parameter, V. Satisfying the above conditions gives $f(r)$ the following form

$$f(r) = \frac{(r-n)}{(r+n)l^2}\,[(r-n)\,(r+3n)+l^2)]. \tag{2.14}$$

Notice here that the roots of $[(r-n)\,(r+3n)+l^2)]$ are all less than n.

Since the fiber has to close smoothly at $r = n$, we do that by setting the period β of the τ direction to be $\beta = \frac{4\pi}{f'(r=n)}$. This implies that

$$\beta = 8\pi\,n \tag{2.15}$$

Notice that the value of the mass parameter that produces the nut in the charged case is identical to that of the uncharged case. Furthermore the stress-energy tensor vanishes upon satisfying the above conditions. As a result the metrics for the two cases in four-dimensions are the identical. It will be clear from the discussion below that this is not the case for higher dimensional solutions. In higher dimensions the mass parameter of the charged case will depend on both the nut charge and the electric charge. This is a new feature that makes higher dimensional charged nut solutions more richer and possess one more parameter, namely q/or V, as it is clear from curvature tensor calculation.

Another feature which is unique to the four-dimensional solution is that the existence of the additional parameter V together with the nut charge induces a magnetic charge and changes electric charge as well even at large radius. As we go to the asymptotical flat limit $n \to 0$ the solution will have no magnetic charge. The magnetic monopole appears as a result of the non-trivial fibration. Also in this limit the electric charge will be only q. This shows that the parameters q and V are not convenient for the four-dimensional solution and they should be replaced by Q and P, the total electric charge and the magnetic charge of the solution as in [9], which shows that this solution is a dyon. This phenomenon does not happen in higher dimensions, i.e. the q will be the total electric charge up to some numerical factor without any contribution from V, but it is clear from the electric field calculation that there is a linear charge density term as well which is proportional to $n^2\,V$. As a result we keep parameterizing these solutions using q and V in the higher dimensional solutions.

2.2 Charged Taub-Bolt-(A)dS Solution

To have a regular bolt solution at the horizon, i.e., at $r = r_b > n$, the following two conditions must be satisfied simultaneously:

[2]In order to have a real one-form A and $f(r)$ upon going to the Lorentzian section one must set $k = 0$

(a) $F(r_b) = 0$
(b) $F'(r_b) = \frac{1}{2n}$
The first condition is just the definition of a horizon and the second is following from the fact that we need to avoid conical singularities at the bolt, keeping at the same time the periodicity of τ to be $8\,\pi\,n$. In addition to these requirements we have another regularity requirement, namely; regularity of the one-form potential A at the bolt $r = r_b$. This leads to the following relation

$$q = -V\,\frac{({r_b}^2 + n^2 + 2\,n\,r_b\,k)}{r_b} \tag{2.16}$$

Imposing condition (a), the mass parameter is given by

$$m = m_b = 1/2\,\frac{[{r_b}^4 + (l^2 - 6\,n^2)\,{r_b}^2 + 4\,l^2\,n^2\,V^2\,(1-k^2) - 3\,n^4 + l^2\,n^2 - 4\,q\,k\,l^2\,V\,n - q^2\,l^2]}{l^2\,r_b} \tag{2.17}$$

By imposing condition (b) one gets r_b as the solution of the following fourth order algebraic equation,

$$6\,{r_b}^4\,n - {r_b}^3\,l^2 + (2\,V^2\,l^2\,n + 2\,n\,l^2 - 6\,n^3)\,{r_b}^2 - 2\,n^3\,V^2\,l^2 = 0 \tag{2.18}$$

One can choose to write the previous expressions in terms of either q or V. Here we have chosen V since it is a bit easier, otherwise the order of the algebraic equation will be higher. One can check numerically that there are solutions (with $r_b > n$)to the above algebraic equation in certain ranges of V and n for a fixed l. But unless we get some expression for the Entropy and the Specific heat we can not estimate the correct ranges of the above parameters and we leave that for a future work.

2.3 Charged Taub-Nut and Taub-Bolt Solution

Here we are presenting the locally asymptotically flat version of the above solutions. The four dimensional solution has the form

$$ds^2 = f(r)\,(d\tau - 2n\cos\theta\,d\phi)^2 + \frac{dr^2}{f(r)} + (r^2 - n^2)(d\theta^2 + sin\theta^2\,d\phi^2) \tag{2.19}$$

where $f(r)$ is given by

$$f(r) = \frac{r^2 - 2m\,r - (q^2 + 4q\,V\,n\,k + 4\,n^2\,V^2\,(k^2-1) - n^2)}{(r^2 - n^2)}. \tag{2.20}$$

The gauge potential has the form

$$A = g(r)\,(dt + 2n\,\cos\theta d\phi). \tag{2.21}$$

Here $g(r)$ is given by

$$g(r) = V\,\frac{r^2 + n^2 + 2\,n\,k\,r}{r^2 - n^2} + \frac{q\,r}{r^2 - n^2}, \tag{2.22}$$

.

As before, in order for this solution to describe a nut solution, we must have $f(r = n) = 0$, so that all of the extra dimensions collapse to zero size. This only happens at specific value of the mass parameter, namely

$$m_n = n \tag{2.23}$$

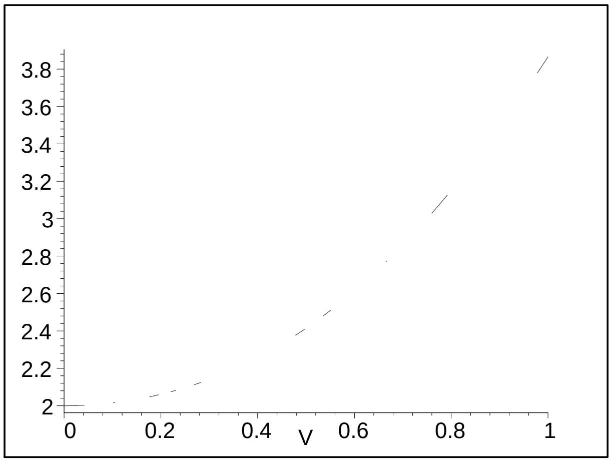

Figure 1: r_b as a function of V, $n = 1$.

otherwise the solution will have a horizon radius $r_h > n$ and will describe a bolt. Choosing the mass parameter to have the above value, $f(r)$ gets the following form

$$f(r) = (r - n)\,(r + n) \tag{2.24}$$

In order to get the fiber to close smoothly at $r = n$, one should set the period β of the τ direction to be $\frac{\beta=4\pi}{f'(r=n)}$. This implies that

$$\beta = 8\pi\, n. \tag{2.25}$$

As we have mentioned the existence of the parameter V together with the nut charge induce a magnetic charge $q \propto V\,n$ which vanishes in the asymptotic flat limit $n \to 0$.

In the case of bolt solution, we impose the regularity condition that the one-form potential A vanishes at the bolt $r = r_b$. It leads to the same relation

$$q = -V\,\frac{({r_b}^2 + n^2 + 2\,n\,r_b\,k)}{r_b} \tag{2.26}$$

Also, imposing condition (a), the mass parameter is given by

$$m = m_b = 1/2\,\frac{[{r_b}^2 + 4\,n^2\,V^2\,(1 - k^2) + n^2 - 4\,q\,k\,V\,n - q^2]}{r_b} \tag{2.27}$$

By imposing condition (b) one gets r_b as the solution of the following fourth order algebraic equation,

$$-\,{r_b}^3 + 2\,n\,(\,V^2 + 1\,)\,{r_b}^2 - 2\,n^3\,V^2 = 0 \tag{2.28}$$

In fact one can solve the above equation analytically but instead of presenting the relatively complicated expression we plotted r_b in 1 as a function of V, where $n = 1$. One can see clearly that $r_b \geq 2n$.

3 Six-Dimensional Solutions

There are two different choices for the base manifold in six dimensions, namely; $\mathbb{CP}^2$ and $S^2 \times S^2$. Let us start with the $\mathbb{CP}^2$ base manifold.

3.1 Charged Taub-nuts-AdS with $\mathcal{B} = \mathbb{CP}^2$

Using $\mathbb{CP}^2$ as a base space, the metric of the charged Taub-NUT-AdS solution has the form

$$ds^2 = f(r)(d\tau + \frac{u^2}{2(1+u^2/6)}(d\psi + \cos\theta d\phi))^2 + f(r)^{-1}dr^2 + (r^2 - n^2)d\Sigma_2{}^2, \tag{3.29}$$

where $d\Sigma_2{}^2$ is the metric over $\mathbb{CP}^2$ which has the following form

$$d\Sigma_2{}^2 = \frac{du^2}{(1+u^2/6)^2} + \frac{u^2}{4(1+u^2/6)^2}(d\psi + \cos\theta d\phi)^2 + \frac{u^2}{4(1+u^2/6)}(d\theta^2 + \sin^2\theta d\phi^2), \tag{3.30}$$

where f(r) is given by

$$\begin{aligned} f(r) = \; & \frac{1}{6\,l^2\,(r^2-n^2)^4}\Big[6\,r^{10} + (2\,l^2 - 42\,n^2)\,r^8 + (156\,n^4 - 16\,n^2\,l^2 - 24\,n^2\,l^2\,V^2)\,r^6 \\ & + (20\,n^4\,l^2 - 180\,n^6 - 360\,n^4\,l^2\,V^2)\,r^4 - 96\,r^3\,n^2\,l^2\,q\,V + (30\,n^8 - 9\,l^2\,q^2 + 216\,n^6\,l^2\,V^2)\,r^2 \\ & + 3\,n^2\,l^2\,q^2 - 6\,n^8\,l^2 + 30\,n^{10} - 216\,n^8\,l^2\,V^2\Big] - \frac{2\,m\,r}{(r^2-n^2)^2} \end{aligned} \tag{3.31}$$

The gauge potential has the form

$$A = g(r)(d\tau + \frac{u^2}{2(1+u^2/6)}(d\psi + \cos\theta d\phi)) \tag{3.32}$$

where $g(r)$ is given by

$$g(r) = q\,\frac{r}{(r^2-n^2)^2} + V\,\frac{-r^4 + 6\,r^2\,n^2 + 3\,n^4}{(r^2-n^2)^2} \tag{3.33}$$

This solution is a generalization to the solutions found in [15, 12]. The thermodynamics for the uncharged version of these solutions have been studied in [14].

For this solution to describe a nut, one must impose $f(r = n) = 0$, this happens when the mass parameter takes the value

$$m_n = \frac{4\,n^3\,(6\,n^2 - l^2 - 9\,l^2\,V^2)}{3\,l^2} \tag{3.34}$$

otherwise the solution will have a horizon radius $r_h > n$ and the solution will describe a bolt. Notice here that the value of the mass that causes the nut is not the same as the one for the uncharged solution, as we mentioned in the previous section, since we have a dependence on either V or q. Choosing the mass parameter to have the above value $f(r)$ will have the following form

$$\begin{aligned} f(r) = \; & \frac{(r-n)}{3\,(n+r)^4\,l^2}\Big[3\,r^5 + 15\,n\,r^4 + (24\,n^2 + l^2)\,r^3 + 5\,n\,l^2\,r^2 \\ & - (27\,n^4 + 12\,n^2\,l^2\,V^2 - 7\,n^2\,l^2)\,r - 15\,n^5 + 3\,l^2\,n^3 + 12\,n^3\,l^2\,V^2\Big] \end{aligned} \tag{3.35}$$

The regularity of the potential A at $r = n$ require that

$$q = -8\,n^3\,V. \tag{3.36}$$

Again, here one can choose to write the previous expressions in terms of either q or V. As before we are going to write expressions in terms of V.

The fiber has to close smoothly at $r = n$, this can be achieved by setting the period β to be

$$\beta = 12\pi\, n. \tag{3.37}$$

In order not to get any solution with $r_b \neq n$ which might spoil the nut solution we must demand that the expression in the square brackets in Eqn. (3.38) does not have any positive roots greater than n. This can be achieved by setting $r = n + x$ in this expression which assumes the form

$$\begin{aligned}&[3\,x^5 + 30\,n\,x^4 + (l^2 + 114\,n^2)\,x^3 + (192\,n^3 + 8\,n\,l^2)\,x^2\\ &+(-12\,n^2\,l^2\,V^2 + 120\,n^4 + 20\,n^2\,l^2)\,x + 16\,l^2\,n^3],\end{aligned} \tag{3.38}$$

then by requiring $|V| < 1/3\,\sqrt{90\,n^2 + 15\,l^2}/l^3$ in this expression, it will have no positive roots. The above requirement might serve as a condition on V for obtaining a nut solution in this case. There is no four dimension analog for this condition. As we have mentioned in the introduction for $\mathcal{B} = \mathbb{CP}^2$ case Taub-Nut solution have no curvature singularities at $r = n$, indeed,

$$R^{ijkl}R_{ijkl} \sim (r+n)^{-12} \tag{3.39}$$

3.2 Charged Taub-Bolt-AdS $\mathbb{CP}^2$ Case

As in Four dimensions, in order to have a regular bolt solution at the horizon, i.e., at $r = r_b > n$, the following two conditions must be satisfied simultaneously:
(a) $F(r_b) = 0$
(b) $F'(r_b) = \frac{1}{3\,n}$
The first condition is just the definition of a horizon and the second is following from the fact that we need to avoid conical singularities at the bolt, keeping at the same time the periodicity of τ to be $12\,\pi\,n$. In addition to these requirements we have another regularity requirement, namely; regularity of the one-form potential A at the bolt $r = r_b$. This leads to the following relation

$$V = -\frac{q\,r_b}{(r_b - n)^2} \tag{3.40}$$

Imposing condition (a), the mass parameter is given by

$$m = m_b = -1/2\,\frac{(-{r_b}^4 + 6\,n^2\,{r_b}^2 - l^2\,{r_b}^2 + 3\,n^4 - l^2\,n^2 + 4\,q\,l^2\,V\,n + q^2\,l^2)}{(l^2\,r_b)} \tag{3.41}$$

By imposing condition (b) together with the regularity of the one-form A one gets r_b as the solution of the following fourth order algebraic equation

$$30\,n\,{r_b}^4 - 2\,{r_b}^3\,l^2 + (6\,n\,l^2 + 27\,n\,l^2\,V^2 - 30\,n^3)\,{r_b}^2 - 27\,n^3\,l^2\,V^2 = 0 \tag{3.42}$$

3.3 Charged Taub-Nut and Bolt for the $\mathbb{CP}^2$ Case

Taking the limit $l \to \infty$ one can obtain the asymptotically locally flat version of the above solutions which represents also new solutions of Einstein-Maxwell equations in six dimensions. Here $f(r)$ takes the following form

$$\begin{aligned}F(r) = \quad & \frac{1}{6\,(r^2-n^2)^4}\,\Big[2\,r^8 - (16\,n^2 + 24\,n^2\,V^2)\,r^6\\ &+(20\,n^4 - 360\,n^4\,V^2)\,r^4 - 96\,r^3\,n^2\,q\,V - (9\,q^2 - 216\,n^6\,V^2)\,r^2\\ &+3\,n^2\,q^2 - 6\,n^8 - 216\,n^8\,V^2\Big] - \frac{2\,m\,r}{(r^2-n^2)^2}\end{aligned} \tag{3.43}$$

[3]This is a more stronger condition but it shows most of the range at which V can produce a nut

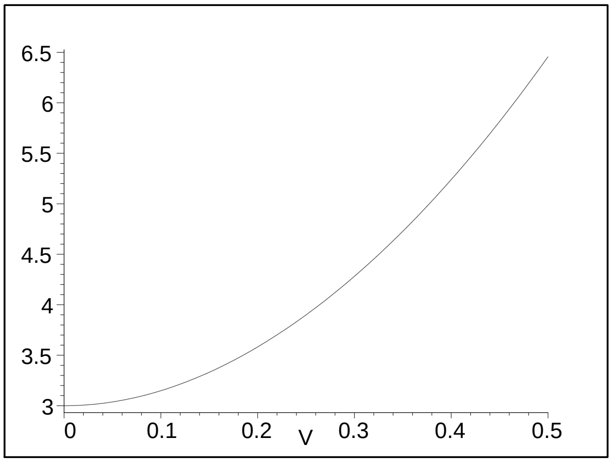

Figure 2: r_b as a function of V, $n = 1$.

The one-form potential A and the function $f(r)$ are the same as the AdS case. For the nut case the mass parameter, m, and the charge, q, must acquire the following form

$$m_n = -\frac{4\,n^3}{3} \tag{3.44}$$

$$q = -8\,n^3\,V \tag{3.45}$$

Imposing the regularity condition for A and shift $r = x + n$, one gets the condition for getting a nut solution

$$|V| < \sqrt{\frac{5}{3}} \tag{3.46}$$

For the Taub-Bolt case imposing the general conditions a) and b) together with the regularity condition on the potential A the horizon radius satisfy the following algebraic equation

$$2\,{r_b}^3 - 3\,n\,(2 + 9\,V^2)\,{r_b}^2 - 27\,n^3\,V^2 = 0 \tag{3.47}$$

Again the analytic solution of this equation is a relatively complicated expression so, instead, we plot r_b in 2 as a function of V, where $n = 1$. One can see clearly that $r_b \geq 3n$.

3.4 Solutions with $\mathcal{B} = S^2 \times S^2$

Using $S^2 \times S^2$ as a base space, the metric of the charged Taub-NUT-AdS solution has the form

$$ds^2 = f(r)(d\tau + 2n\cos\theta_1 d\phi_1 + 2n\cos\theta_2 d\phi_2)^2 + f(r)^{-1}dr^2 + (r^2 - n^2)({d\theta_1}^2 + {\sin\theta_1}^2 d\phi_1^2 + {d\theta_2}^2 + {\sin\theta_2}^2 d\phi_2^2) \tag{3.48}$$

$$A = g(r)(d\tau + 2\,n\,\cos\theta_1 d\phi_1 + 2\,n\,\cos\theta_2 d\phi_2) \tag{3.49}$$

where $g(r)$ is given by

$$g(r) = q\,\frac{r}{(r^2 - n^2)^2} + V\,\frac{-r^4 + 6\,r^2\,n^2 + 3\,n^4}{(r^2 - n^2)^2} \tag{3.50}$$

Here $f(r)$ has the same form as the one for $\mathcal{B} = \mathbb{CP}^2$ case. This Six dimensional solution is a generalization to the solution found in [15, 12]. Another exotic form similar to these metrics has been found in [18]. Also, these uncharged solutions have been used to find some bubble solutions in [17]. For $\mathcal{B} = S^2 \times S^2$ case Taub-Nut solution have curvature singularities at $r = n$, indeed,

$$R^{ijkl}R_{ijkl} \sim (r-n)^{-2} \tag{3.51}$$

But the Taub-Bolt cases for such solution do not suffer from such a problem since the singularity is hidden behind the horizon.

4 Final Remarks

We have presented a class of d–dimensions nut-charged solutions for Einstein-Maxwell field equations. These solutions depend on two extra parameters other than the mass and the nut charge, namely; the charge q and the potential at infinity V. The existence of the parameter V enables us to get a regularity condition on the one-form potential which we found identical to that required to obtain a nut solution. The nut conditions show that the mass not only depends on the nut charge as in four dimensions but also on the charge/or the potential V as a result we get a smooth six dimensional manifold for $\mathcal{B} = \mathbb{CP}^2$. One might be concerned by the existence of naked singularities for the $\mathcal{B} = S^2 \times S^2$ case. However, one would argue that from the point of view of the adS/CFT duality that it is possible to resolve these singularities, for example, using Gubser's [21] criterion that these are 'good' singularities, in the sense that a finite temperature deformation will yield a non-singular solution which is the bolt. It will be interesting to probe the physics of these new smooth gravitational solutions especially in the context of AdS/CFT correspondence and their thermodynamics. One can generalize these constructions easily to eight and ten dimensions. It is interesting also to generalization this construction to include black holes with topological horizons and study the nut and bolt conditions.

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